

## **UNIVERSITY** of LIMERICK

## OLLSCOIL LUIMNIGH

## FACULTY OF SCIENCE AND ENGINEERING

## DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA 4005SEMESTER: Autumn 2011MODULE TITLE: Engineering Maths T1DURATION OF EXAMINATION: 2hrs 30minsLECTURER: Dr. William LeePERCENTAGE OF TOTAL MARKS: 80%

INSTRUCTIONS TO CANDIDATES: Answer any 5 questions. All questions carry equal marks. Full marks for correct answers to any 5 questions. Open book exam. Engineering Maths T1

1. Find all first order and second order partial derivatives of the following functions:

(a) 
$$f_1(x,y) = 5x^2 + 4y^3 + xy$$

(b) 
$$f_2(x,y) = (x+y)\sin^2(x+y)$$

(c) 
$$f_3(x, y, z) = \exp(x^2 + xy + y^2 - yz + z^2)$$

2. The rise velocity, v of a small gas bubble in a liquid is given by

$$v = \frac{2r^2\rho g}{9\mu}$$

where r is the radius of the bubble,  $\rho = 10^3 \,\mathrm{kg}\,\mathrm{m}^{-3}$  is difference in density between the liquid and the gas,  $g = 9.81 \,\mathrm{m}\,\mathrm{s}^{-2}$  is the acceleration due to gravity, and  $\mu$  is the viscosity of the liquid.

- (a) Rearrange the equation to get an equation for the viscosity  $\mu$ .
- (b) Write the total differential of  $\mu$  treating  $\rho$  and g as constants and r and v as variables.
- (c) Write an expression for the maximum error in  $\mu$  in terms of r, v, and their uncertainties,  $\delta r$  and  $\delta v$  (assuming those uncertainties to be small).
- (d) Measurements of r and v give  $r = 1 \text{ mm} \pm 0.1 \text{ mm}$  and  $v = 10 \text{ mm s}^{-1} \pm 0.5 \text{ mm s}^{-1}$ . Calculate the estimated value of  $\mu$  and its maximum error. (SI units of viscosity are Pa s.)

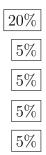


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- 3. Calculate the following indefinite integrals.
  - (a)  $\int \left(4x^3 e^x + \frac{2}{x}\right) dx$

(b) 
$$\int \frac{x^3}{x^4+1} \,\mathrm{d}x$$

- (c)  $\int x e^{-2x} dx$
- (d)  $\int x\sqrt{x^2-1} \,\mathrm{d}x$



- 4. Consider the curve  $y = \cosh(x)$  between x = 0 and x = 2. Use definite integrals to find:
  - (a) The length of the curve.
  - (b) Find the area between the curve and the x axis.
  - (c) Find the volume generated when this area is rotated about the x axis.
- 5. Find the general solution of the differential equations (a prime, ', denotes differentiation with respect to x:  $y' = \frac{dy}{dx}$ )
  - (a) y' y = 0
  - (b)  $y' + \cos(x) y = 0$

(c) 
$$2y'' + y' + 4y = 1$$

- 6. Use Laplace transforms to solve the differential equations (a prime, ', denotes differentiation with respect to x:  $y' = \frac{dy}{dx}$ )
  - (a) y' + 3y = 7, y(0) = 1.

(b) 
$$y' - 3y = 4$$
,  $y(0) = -1$ .

(c) 3y'' + 6y' + 5y = 0, y(0) = 1, y'(0) = -1.

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